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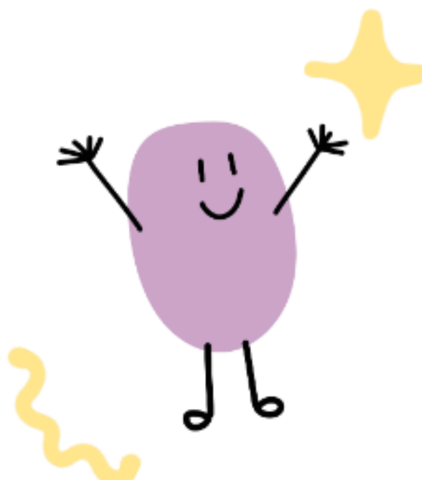
H2 Mathematics



Functions Package

“Believe you can and you’re already halfway there.” – Theodore Roosevelt

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INTRODUCTION: What is a function?

Functions look like this: $\mathbf{f}: \boldsymbol{x} \rightarrow$ ______ or $\mathbf{f}(\boldsymbol{x}) =$ ______ , $\boldsymbol{x} \in$ ______

For example, 1) $\mathrm{f}: x \rightarrow x^2 + 2\,,\ x \in \mathbb{R}$ **OR**

2) $\mathrm{f}(x) \rightarrow x^2 + 2\,,\ x \in \mathbb{R}\,, x > 0$

<u>**CONCEPT #1:**</u> **DOMAIN AND RANGE**

The **domain** of a function f, denoted by **Df**, is usually found after the function is stated. This represents the range of values of x.

- ⇒ i.e. for the example of $\mathrm{f}(x) \rightarrow x^2 + 2\,,\ x \in \mathbb{R}\,, x > 0$, Df = (0 , ∞)
- ⇒ **TEST**: What is the domain of this function: $\mathrm{f}(x) \rightarrow x^3\,,\ x \in \mathbb{R}\,, x \leq 2$?

The **range** of a function f, denoted by **Rf**, is the set of answers for f(x) OR the set of values of y. To find Rf, we often sketch a graph of $y = \mathrm{f}(x)$.

- ⇒ i.e. for $\mathrm{f}(x) \rightarrow x^2 + 2\,,\ x \in \mathbb{R}\,, x > 0$, we draw it's graph and find out what values y can take:

Example 1: Draw the graphs for the following functions and state its range:

a) $\mathrm{f}(x) = \ln(x+1)\ , x \in \mathbb{R}\,, x > 1$	**b)** $\mathrm{g}(x) = \begin{cases} 3 - x^2\,, & x \geq 0 \\ x + 2\,, & x < 0 \end{cases}$

CONCEPT #2: INVERSE FUNCTIONS

An inverse function looks like this: **$f^{-1}(x)$ =** ______

DOMAIN AND RANGE OF AN INVERSE FUNCTION:

Domain: $D_{f-1} = R_f$ **Range: $R_{f-1} = D_f$**	

However, not every function has an inverse. For an inverse to exist, the function must: be a **one-one** function.

A function f is only one-one if every horizontal line $y = k$, $k \in R$f cuts the graph of f at **one point** only.

<u>**How to prove this?**</u>

1. Draw the graph of f(x)
2. Draw a horizontal line y = k which cuts the graph at one/ more than one point
3. State your conclusion
   a. Since every horizontal line $y = k$, $k \in R$f cuts the graph at one point only, f is a one-one function and f-1 exists
   b. Vice versa

FINDING F^{-1}

It's simple.

<u>Step 1:</u> Let f(x) = y

<u>Step 2:</u> Make x the subject (i.e. express x in terms of y)

<u>Step 3:</u> The expression of x in terms of y is the expression for f-1, simply switch y to x.

For instance, let us find the inverse of f(x) = x + 3.

Step 1) Let y = x + 3 , then Step 2) x = y – 3 , and Step 3) $f^{-1}(x)$ = x – 3 . That's it!

GRAPHICAL RELATIONSHIP BETWEEN A FUNCTION AND ITS INVERSE

Memorise:

The graph of $y = f(x)$ and $y = f^{-1}(x)$ are **reflections** of each other about the line $y = x$.

The steps to drawing an inverse graph:

1. Use the same scale for both the x and y axes.
2. Draw the graph of $y = f(x)$. [You can use the GC to help you with this]
3. Draw the line of $y = x$ in dotted lines as a guideline
4. Draw the graph of $y = f^{-1}(x)$ by reflecting the f(x) graph about the line $y = x$.
    a. To help you with this step, you can:
        i. Plot the points of intersection of the f(x) graph with the $y = x$ line
        ii. Turn the paper around
        iii. Flip any known coordinates and plot (i.e. (1, 2) point becomes (2, 1)
    b. Check the inverse graph shape using your GC [draw DrawInv Y1* enter]

COMPOSITE FUNCTIONS

A composite function is the combination of two functions, such that the output of one function is used as the input of another. It looks something like this: **gf(x)** where the **second letter f, is mapped first**; ie. gf(x) = g[f(x)]

Note:

I. In general, $gf(x) \neq fg(x)$
II. The composite function ff(x) is written as $f^2(x)$, and fff(x) as $f^3(x)$, and so on

DOMAIN AND RANGE OF AN INVERSE FUNCTION:

Domain: $D_{gf} = D_f$ **Range: * SPECIAL**	

EXISTENCE OF A COMPOSITE FUNCTION

Just like what we learnt with an inverse function, not all composite functions exist. The composite function gf only exists when:

$$R_f \subseteq D_g$$

SPECIAL CASE: Composition of a function and its inverse:

$$ff^{-1}(x) = x$$